PROBLEM-SOLVING NOTE

Choice of Coordinate System: Incline

On an incline, align one axis (x) parallel to the surface, and the other axis (y) perpendicular to the surface. That way the motion is in the x direction. Since no motion occurs in the y direction, we know that $a_y = 0$.

a larger area of contact doesn't produce a larger force? One way to think about this is to consider that when the area of contact is large, the normal force is spread out over a large area, giving a small force per area, F/A. As a result, the microscopic hills and valleys are not pressed too deeply against one another. On the other hand, if the area is small, the normal force is concentrated in a small region, which presses the surfaces together more firmly, due to the large force per area. The net effect is roughly the same in either case.

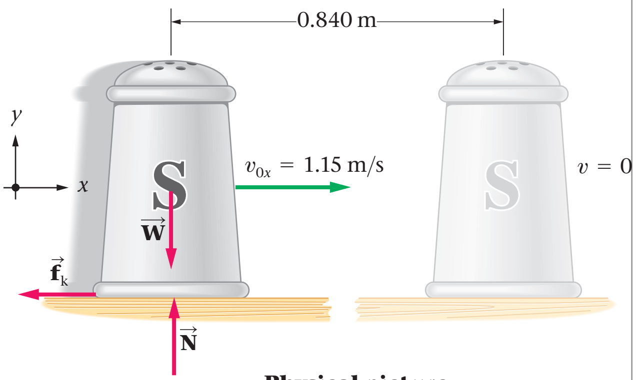
The next Example considers a commonly encountered situation in which kinetic friction plays a decisive role.

EXAMPLE 6-1 PASS THE SALT—PLEASE

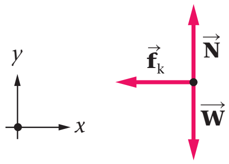
Someone at the other end of the table asks you to pass the salt. Feeling quite dashing, you slide the 50.0-g salt shaker in that direction, giving it an initial speed of 1.15 m/s. **(a)** If the shaker comes to rest with constant acceleration in 0.840 m, what is the coefficient of kinetic friction between the shaker and the table? **(b)** How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32 m/s?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion, and the positive y direction to be upward. Two forces act in the y direction: the shaker's weight, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$, and the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$. Only one force acts in the x direction: the force of kinetic friction, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$. Notice that the shaker moves through a distance of 0.840 m with an initial speed $v_{0x} = 1.15$ m/s.



Physical picture



Free-body diagram

REASONING AND STRATEGY

a. The frictional force has a magnitude of $f_k = \mu_k N$, and hence it follows that $\mu_k = f_k/N$. Therefore, we need to find the magnitudes of the frictional force, f_k, and the normal force, N, to determine μ_k. To find f_k we set $\sum F_x = ma_x$, and find a_x with the kinematic equation $v_x^2 = v_{0x}^2 + 2a_x\Delta x$. To find N we set $a_y = 0$ (since there is no motion in the y direction) and solve for N using $\sum F_y = ma_y = 0$.

b. The coefficient of kinetic friction is independent of the sliding speed, and hence the acceleration of the shaker is also independent of the speed. As a result, we can use the acceleration from part (a) in the equation $v_x = v_{0x} + a_x t$ to find the sliding time.

Known Mass of salt shaker, $m = 50.0$ g; initial speed, $v_0 = 1.15$ m/s or 1.32 m/s; sliding distance, $\Delta x = 0.840$ m.

Unknown (a) Coefficient of kinetic friction, $\mu_k = ?$ **(b)** Time to come to rest, $t = ?$

SOLUTION

Part (a)

1. Set $\sum F_x = ma_x$ to find f_k in terms of a_x:

$$\sum F_x = -f_k = ma_x \quad \text{or} \quad f_k = -ma_x$$

2. Determine a_x by using the kinematic equation relating velocity to position, $v_x^2 = v_{0x}^2 + 2a_x\Delta x$:

$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$

$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0 - (1.15\ \text{m/s})^2}{2(0.840\ \text{m})} = -0.787\ \text{m/s}^2$$

3. Set $\sum F_y = ma_y = 0$ to find the normal force, N:

$$\sum F_y = N + (-W) = ma_y = 0 \quad \text{or} \quad N = W = mg$$

4. Substitute $N = mg$ and $f_k = -ma_x$ (with $a_x = -0.787\ \text{m/s}^2$) into $\mu_k = f_k/N$ to find μ_k:

$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787\ \text{m/s}^2)}{9.81\ \text{m/s}^2} = 0.0802$$

Part (b)

5. Use $a_x = -0.787\ \text{m/s}^2$, $v_{0x} = 1.32$ m/s, and $v_x = 0$ in $v_x = v_{0x} + a_x t$ to solve for the time, t:

$$v_x = v_{0x} + a_x t \quad \text{or}$$

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - (1.32\ \text{m/s})}{-0.787\ \text{m/s}^2} = 1.68\ \text{s}$$

INSIGHT

Notice that m canceled in Step 4, so our result for the coefficient of friction is independent of the shaker's mass. For example, if we were to slide a shaker with twice the mass, but with the same initial speed, it would slide the same distance. It's unlikely this independence would have been apparent if we had worked the problem numerically rather than symbolically. Part (b) shows that the same comments apply to the sliding time—it too is independent of the shaker's mass.

CONTINUED

PRACTICE PROBLEM

Given an initial speed of 1.15 m/s and a coefficient of kinetic friction equal to 0.120, what are **(a)** the acceleration of the shaker and **(b)** the distance it slides? [**Answer: (a)** $a_x = -1.18\ \text{m/s}^2$, **(b)** 0.560 m]

Some related homework problems: Problem 3, Problem 13

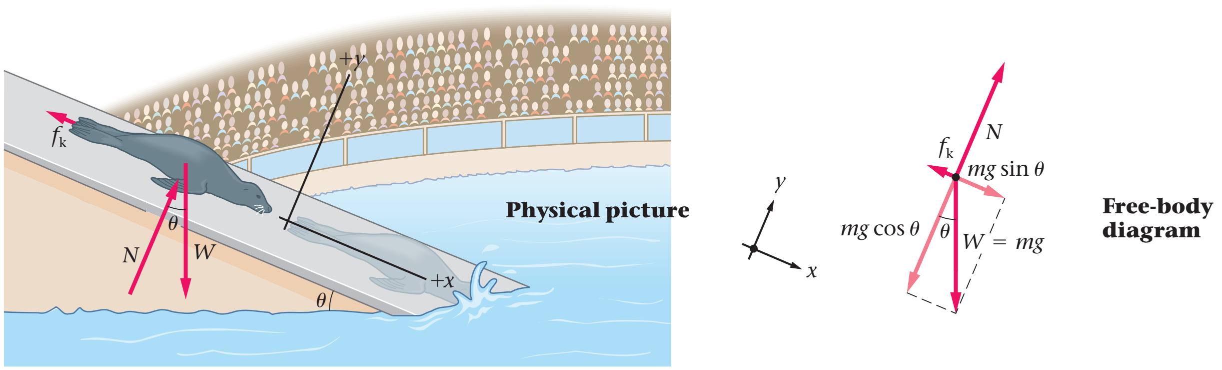
In the next Example, the system is inclined at an angle θ relative to the horizontal. As a result, the normal force responsible for the kinetic friction is *less* than the weight of the object.

EXAMPLE 6-2 MAKING A BIG SPLASH

A trained sea lion slides from rest with constant acceleration down a 3.0-m-long ramp into a pool of water. If the ramp is inclined at an angle of $\theta = 23°$ above the horizontal and the coefficient of kinetic friction between the sea lion and the ramp is 0.26, how much time does it take for the sea lion to make a splash in the pool?

PICTURE THE PROBLEM

As is usual with inclined surfaces, we choose one axis to be parallel to the surface and the other to be perpendicular to it. In our sketch, the sea lion accelerates in the positive x direction ($a_x > 0$), having started from rest, $v_{0x} = 0$. We are free to choose the initial position of the sea lion to be $x_0 = 0$. There is no motion in the y direction, and therefore $a_y = 0$. Finally, we note from the free-body diagram that $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$, and $\vec{\mathbf{W}} = (mg\sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



REASONING AND STRATEGY

We can use the kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to find the time of the sea lion's slide. It will be necessary, however, to first determine the acceleration of the sea lion in the x direction, a_x.

To find a_x we apply Newton's second law to the sea lion. First, we can find N by setting $\sum F_y = ma_y$ equal to zero (since $a_y = 0$). It is important to start by finding N because we need it to find the force of kinetic friction, $f_k = \mu_k N$. Using f_k in the sum of forces in the x direction, $\sum F_x = ma_x$, allows us to solve for a_x and, finally, for the time.

Known Length of ramp, $x = 3.0$ m; angle of incline, $\theta = 23°$; coefficient of kinetic friction, $\mu_k = 0.26$.
Unknown Sliding time, $t = ?$

SOLUTION

1. We begin by resolving each of the three force vectors into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{k,x} = -f_k = -\mu_k N \qquad f_{k,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\sum F_y = ma_y = 0$ to find N. We see that N is less than the weight, mg:

$$\sum F_y = N - mg\cos\theta = ma_y = 0$$
$$N = mg\cos\theta$$

3. Next, set $\sum F_x = ma_x$. Notice that the mass cancels in this equation:

$$\sum F_x = mg\sin\theta - f_k = mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

4. Solve for the acceleration in the x direction, a_x:

$$a_x = g(\sin\theta - \mu_k\cos\theta) = (9.81\ \text{m/s}^2)[\sin 23° - (0.26)\cos 23°] = 1.5\ \text{m/s}^2$$

CONTINUED

Known Symbolic masses of the blocks, m_1, m_2.
Unknown **(a)** Acceleration of the blocks, $a = ?$ **(b)** Tension in the string, $T = ?$

SOLUTION

Part (a)

1. First, write $\sum F_{1,x} = m_1 a$. Note that the only force acting on m_1 in the x direction is T:
$$\sum F_{1,x} = T = m_1 a$$
$$T = m_1 a$$
2. Next, write $\sum F_{2,x} = m_2 a$. In this case, two forces act in the x direction: $W_2 = m_2 g$ (positive direction) and T (negative direction):
$$\sum F_{2,x} = m_2 g - T = m_2 a$$
$$m_2 g - T = m_2 a$$
3. Sum the two relationships obtained to eliminate T:
$$T = m_1 a$$
$$\underline{m_2 g - T = m_2 a}$$
$$m_2 g = (m_1 + m_2)a$$
4. Solve for a:
$$a = \left(\frac{m_2}{m_1 + m_2}\right)g$$

Part (b)

5. Substitute a into the first equation $(T = m_1 a)$ to find T:
$$T = m_1 a = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g$$

INSIGHT

We could just as well have determined T using $m_2 g - T = m_2 a$, though the algebra is a bit messier. Also, notice that $a = 0$ if $m_2 = 0$, and that $a = g$ if $m_1 = 0$, as expected. Similarly, $T = 0$ if either m_1 or m_2 is zero. This type of check, where you connect equations with physical situations, is one of the best ways to increase your understanding of physics.

PRACTICE PROBLEM

Find the acceleration and tension for the case $m_1 = 1.50$ kg and $m_2 = 0.750$ kg, and compare the tension to $m_2 g$.
[**Answer:** $a = 3.27\ \text{m/s}^2$, $T = 4.91\ \text{N} < m_2 g = 7.36\ \text{N}$]

Some related homework problems: Problem 38, Problem 41

Conceptual Example 6-12 shows that the tension in the string is less than $m_2 g$. Let's rewrite our solution for T from Example 6-13 to show that this is indeed the case. We have

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g = \left(\frac{m_1}{m_1 + m_2}\right)m_2 g$$

Noting that the ratio $m_1/(m_1 + m_2)$ is always less than 1 (as long as m_2 is nonzero), it follows that $T < m_2 g$, as expected.

A Method for Measuring the Acceleration due to Gravity We conclude this section with a classic system that can be used to measure the acceleration due to gravity. It is referred to as Atwood's machine, and it is basically two blocks of different mass connected by a string that passes over a pulley. The resulting acceleration of the blocks is related to the acceleration due to gravity by a relatively simple expression, which we derive in the following Example.

EXAMPLE 6-14 ATWOOD'S MACHINE

Atwood's machine consists of two masses connected by a string that passes over a pulley, as shown in the sketch. Find the acceleration of the masses for general m_1 and m_2, and evaluate for the specific case $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.

PICTURE THE PROBLEM

Our sketch shows Atwood's machine, along with our choice of coordinate directions for the two blocks. Note that both blocks accelerate in the positive x direction with accelerations of equal magnitude, a. From the free-body diagrams we can see that for mass 1 the weight is in the negative x direction and the tension is in the positive x direction. For mass 2, the tension is in the negative x direction and the weight is in the positive x direction. The tension has the same magnitude T for both masses, but their weights are different.

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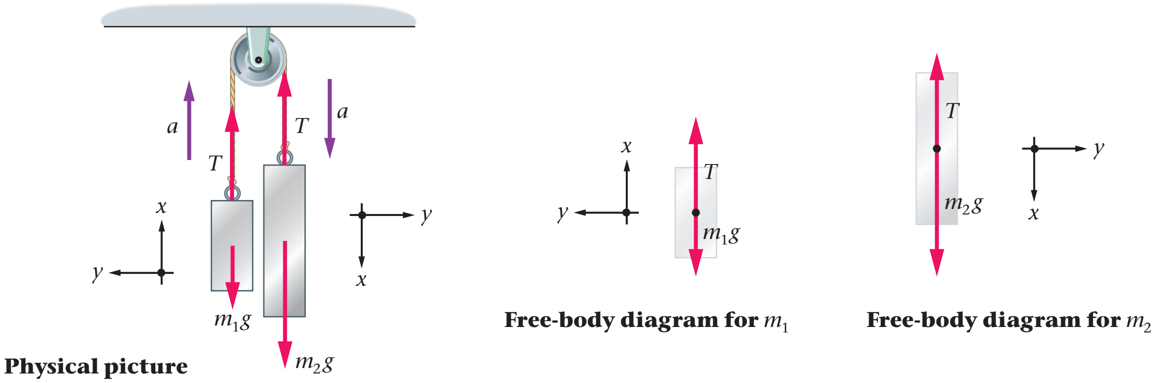


REASONING AND STRATEGY

To find the acceleration of the blocks, we follow the same strategy given in the previous Example. In particular, we start by applying Newton's second law to each block individually, using the fact that $a_{1,x} = a_{2,x} = a$. This gives two equations, both involving the tension T and the acceleration a. Eliminating T allows us to solve for the acceleration.

Known Masses, $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.
Unknown Acceleration of the masses, $a = ?$

SOLUTION

1. Begin by writing out the expression $\sum F_{1,x} = m_1 a$. Note that two forces act in the x direction: T (positive direction) and $m_1 g$ (negative direction):

$$\sum F_{1,x} = T - m_1 g = m_1 a$$

2. Next, write out $\sum F_{2,x} = m_2 a$. The two forces acting in the x direction in this case are $m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

3. Sum the two relationships obtained above to eliminate T:

$$\begin{array}{r} T - m_1 g = m_1 a \\ m_2 g - T = m_2 a \\ \hline (m_2 - m_1)g = (m_1 + m_2)a \end{array}$$

4. Solve for a:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

5. To evaluate the acceleration, substitute numerical values for the masses and for g:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$
$$= \left(\frac{4.4\text{ kg} - 3.1\text{ kg}}{3.1\text{ kg} + 4.4\text{ kg}}\right)(9.81\text{ m/s}^2) = 1.7\text{ m/s}^2$$

INSIGHT

Because m_2 is greater than m_1, we find that the acceleration is positive, meaning that the masses accelerate in the positive x direction. On the other hand, if m_1 were greater than m_2, we would find that a is negative, indicating that the masses accelerate in the negative x direction. Finally, if $m_1 = m_2$, we have $a = 0$, as expected.

PRACTICE PROBLEM — PREDICT/CALCULATE

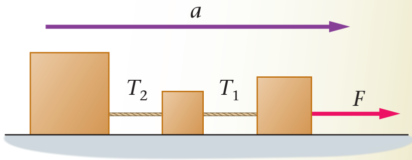
(a) If m_1 is increased by a small amount, does the acceleration of the blocks increase, decrease, or stay the same? Explain. **(b)** Check your answer to part (a) by evaluating the acceleration for $m_1 = 3.3$ kg. [**Answer: (a)** The acceleration decreases because the masses are more nearly balanced. **(b)** $a = 1.4\text{ m/s}^2$]

Some related homework problems: Problem 40, Problem 41, Problem 43

Enhance Your Understanding

(Answers given at the end of the chapter)

4. Three boxes are connected by ropes and pulled across a smooth, horizontal surface with an acceleration a, as shown in FIGURE 6-21. The force applied to the first box on the right is F, the tension in the rope connecting the first



▲ **FIGURE 6-21** Enhance Your Understanding 4

CONTINUED

and second boxes is T_1, and the tension in the rope connecting the second and third boxes is T_2. Rank the three forces (F, T_1, T_2) in order of increasing magnitude. Indicate ties where appropriate.

Section Review

- Objects connected by strings have the same acceleration. This implies certain values for the tension in the strings.

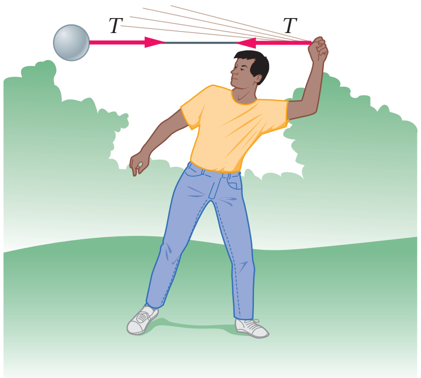
Big Idea 4 When an object moves in a circular path, it accelerates toward the center of the path. As a result, circular motion requires a force directed toward the center.

6-5 Circular Motion

According to Newton's second law, if no force acts on an object, it will move with constant speed in a constant direction. A force is required to change the speed, the direction, or both. For example, if you drive a car with constant speed on a circular track, the direction of the car's motion changes continuously. A force must act on the car to cause this change in direction. We would like to know two things about a force that causes circular motion: (i) What is its direction, and (ii) What is its magnitude?

First, let's consider the direction of the force. Imagine swinging a ball tied to a string in a circle above your head, as shown in **FIGURE 6-22**. As you swing the ball, you feel a tension in the string pulling outward. Of course, on the other end of the string, where it attaches to the ball, the tension pulls inward, toward the center of the circle. Thus, the force the ball experiences is a force that is always directed toward the center of the circle. In summary:

> To make an object move in a circle with constant speed, a force must act on it that is directed toward the center of the circle.



▲ **FIGURE 6-22 Swinging a ball in a circle** The tension in the string pulls outward on the person's hand and pulls inward on the ball.

Because the ball is acted on by a *force* toward the center of the circle, it follows that it must be *accelerating* toward the center of the circle. This might seem odd at first: How can a ball that moves with constant speed have an acceleration? The answer is that acceleration is produced whenever the speed or direction of the velocity changes—and in circular motion, the direction changes continuously. The resulting center-directed acceleration is called **centripetal acceleration** (centripetal is from the Latin for "center seeking").

Ball Leaves Circular Track

Calculating the Centripetal Acceleration Let's calculate the magnitude of the centripetal acceleration, a_{cp}, for an object moving with a constant speed v in a circle of radius r. **FIGURE 6-23** shows the circular path of an object, with the center of the circle at the origin. To calculate the acceleration at the top of the circle, at point P, we first calculate the average acceleration from point 1 to point 2:

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} \qquad \text{6-10}$$

The instantaneous acceleration at P is the limit of $\vec{\mathbf{a}}_{av}$ as points 1 and 2 move closer to P.

Referring to Figure 6-23, we see that $\vec{\mathbf{v}}_1$ is at an angle θ above the horizontal, and $\vec{\mathbf{v}}_2$ is at an angle θ below the horizontal. Both $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ have a magnitude v. Therefore, we can write the two velocities in vector form as follows:

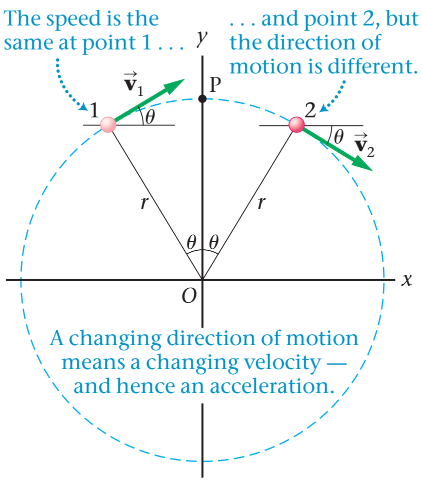
$$\vec{\mathbf{v}}_1 = (v\cos\theta)\hat{\mathbf{x}} + (v\sin\theta)\hat{\mathbf{y}}$$
$$\vec{\mathbf{v}}_2 = (v\cos\theta)\hat{\mathbf{x}} + (-v\sin\theta)\hat{\mathbf{y}}$$

Substituting these results into $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{-2v\sin\theta}{\Delta t}\hat{\mathbf{y}} \qquad \text{6-11}$$

Notice that $\vec{\mathbf{a}}_{av}$ points in the negative y direction—which, at point P, is toward the center of the circle.

To complete the calculation, we need to know the time, Δt, it takes the object to go from point 1 to point 2. Since the object's speed is v, and the distance from point



▲ **FIGURE 6-23 A particle moving with constant speed in a circular path centered on the origin** The speed of the particle is constant, but its velocity is constantly changing direction. Because the velocity changes, the particle is accelerating.

TABLE 6-2 $\frac{\sin\theta}{\theta}$ **for Values of** θ **Approaching Zero**

θ, radians	$\frac{\sin\theta}{\theta}$
1.00	0.841
0.500	0.959
0.250	0.990
0.125	0.997
0.0625	0.999

PHYSICS IN CONTEXT
Looking Back

The derivation of the direction and magnitude of centripetal acceleration makes extensive use of our knowledge of vectors from Chapter 3, and especially how to resolve vectors into components.

PHYSICS IN CONTEXT
Looking Ahead

Circular motion comes up again in a number of physical systems, especially when we consider gravitational orbital motion in Chapter 12 and the Bohr model of the hydrogen atom in Chapter 31.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Circular Motion

In circular motion, it is convenient to choose the coordinate system so that one axis points toward the center of the circle. Then, we know that the acceleration in that direction must be $a_{cp} = v^2/r$.

1 to point 2 is $d = r(2\theta)$, where θ is measured in radians (see Appendix, page A-2 for a discussion of radians and degrees), we find

$$\Delta t = \frac{d}{v} = \frac{2r\theta}{v} \qquad \text{6-12}$$

Combining this result for Δt with the previous result for $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{-2v\sin\theta}{(2r\theta/v)}\hat{\mathbf{y}} = -\frac{v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{\mathbf{y}} \qquad \text{6-13}$$

To find $\vec{\mathbf{a}}$ at point P, we let points 1 and 2 approach P, which means letting θ go to zero. Table 6-2 shows that as θ goes to zero $(\theta \to 0)$, the ratio $(\sin\theta)/\theta$ goes to 1:

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

It follows, then, that the instantaneous acceleration at point P is

$$\vec{\mathbf{a}} = -\frac{v^2}{r}\hat{\mathbf{y}} = -a_{cp}\hat{\mathbf{y}} \qquad \text{6-14}$$

As mentioned, the direction of the acceleration is toward the center of the circle, and now we see that its magnitude is

$$a_{cp} = \frac{v^2}{r} \qquad \text{6-15}$$

We can summarize these results as follows:

- When an object moves in a circular path of radius r with constant speed v, its centripetal acceleration has a magnitude given by $a_{cp} = v^2/r$.
- A centripetal force must be applied to an object to give it circular motion. For an object of mass m, the net force acting on it must have a magnitude given by

$$f_{cp} = ma_{cp} = m\frac{v^2}{r} \qquad \text{6-16}$$

This force must be directed toward the center of the circle.

The **centripetal force,** f_{cp}, can be produced in any number of ways. For example, f_{cp} might be the tension in a string, as in the example with the ball, or it might be due to friction between tires and the road, as when a car turns a corner. In addition, f_{cp} could be the force of gravity that causes a satellite, or the Moon, to orbit the Earth. Thus, f_{cp} is a force that must be present to cause circular motion, but the specific cause of f_{cp} varies from system to system. The centripetal force in a carnival ride is illustrated in FIGURE 6-24.

We now show how these results for centripetal force and centripetal acceleration can be applied in practice.

◀ **FIGURE 6-24** The people enjoying this carnival ride are experiencing a centripetal acceleration of roughly 10 m/s^2 directed inward, toward the axis of rotation. The force needed to produce this acceleration, which keeps the riders moving in a circular path, is provided by the horizontal component of the tension in the chains.

EXAMPLE 6-15 ROUNDING A CORNER

A 1200-kg car rounds a corner of radius $r = 45$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.82$, what is the greatest speed the car can have in the corner without skidding?

PICTURE THE PROBLEM

In the first sketch we show a bird's-eye view of the car as it moves along its circular path. The next sketch shows the car moving directly toward the observer. Notice that we have chosen the positive x direction to point toward the center of the circular path, and the positive y axis to point vertically upward. We also indicate the three forces acting on the car: gravity, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$; the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$; and the force of static friction, $\vec{\mathbf{f}}_s = \mu_s N\hat{\mathbf{x}}$.

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